2018-2019 Guide

April 29– End of School Year

Eureka

Module 7: *Geometry and Measurement Word Problems*

ORANGE PUBLIC SCHOOLS

OFFICE OF CURRICULUM AND INSTRUCTION

OFFICE OF MATHEMATICS

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Module 7 Performance Overview

- This module offers students intensive practice with word problems, as well as hands-on investigation experiences with geometry and perimeter. The module begins with solving one- and two-step word problems based on a variety of topics studied throughout the year, using all four operations. Line plots help students draw conclusions about perimeter and area measurements. Students solve word problems involving area and perimeter using all four operations.

- Topic A emphasizes modeling and reasoning to develop ways to solve a problem. They incorporate opportunities for students to independently work through a problem and persevere through doing so.

- Topic B introduces an exploration of geometry. Students build on ideas about polygons and their properties, specifically developing and expanding their knowledge of quadrilaterals. They explore the attributes of quadrilaterals and classify examples into various categories, including recognizing the characteristics of polygons.

- As they learn about perimeter as an attribute of plane figures, students apply their knowledge to real world situations through problem solving. They measure side lengths of shapes in whole number units to determine perimeter and solve problems where side lengths are given. Students consider the efficiency of their strategies and identify tools for solving; for example, they use multiplication as a tool when measurements are repeated.

- Topic D utilizes the line plot to help students draw conclusions about perimeter and area measurements. As they move through the lessons in this topic, students notice and compare differences in the strategies for finding area when given a perimeter and for finding perimeter given an area. By the end of the topic they are able to conclude that there is no direct relationship between area and perimeter, meaning that if an area is given there is no way of knowing a shape's corresponding perimeter.

- In Topic E, students solve problems involving area and perimeter. Given specific perimeter measurements, they reason about the different side lengths that may be produced. Students compare and analyze their work, discussing how different choices for side lengths can affect area while conforming to the criteria for perimeter.

- Topic F concludes the school year with a set of engaging lessons that briefly review the fundamental Grade 3 concepts of fractions, multiplication, and division. The final lessons in this topic are fluency based and engage students in games that provide practice to solidify their proficiency with Grade 3 skills. Using simple origami techniques, they create booklets of these games. The booklets go home and become resources for summer practice.

Module 7: *Geometry and Measurement Word Problems*

Pacing:		
April 29 – End of School Year		
38 Days		
Topic	**Lesson**	**Lesson Objective/ Supportive Videos**
Topic A: Solving Word Problems	Lesson 1	Solve word problems in varied contexts using a letter to represent the unknown. https://www.youtube.com/watch?v
	Lesson 2	Solve word problems in varied contexts using a letter to represent the unknown. https://www.youtube.com/watch?v
	Lesson 3	Share and critique peer solution strategies to varied word problems. https://www.youtube.com/watch?v
Topic B: Attributes of Two-Dimensional Figures	Lesson 4	Compare and classify quadrilaterals. https://www.youtube.com/watch?v
	Lesson 5	Compare and classify other polygons. https://www.youtube.com/watch?v
	Lesson 6	Draw polygons with specified attributes to solve problems. https://www.youtube.com/watch?v
	Lesson 7	Reason about composing and decomposing polygons using tetrominoes. https://www.youtube.com/watch?v
	Lesson 8	Create a tangram puzzle and observe relationships among the shapes. https://www.youtube.com/watch?v
Topic C: Problem Solving with Perimeter	Lesson 10	Decompose quadrilaterals to understand perimeter as the boundary of a shape. https://www.youtube.com/watch?v
	Lesson 12	Measure side lengths in whole number units to determine the perimeter of polygons. https://www.youtube.com/watch?v
	Lesson 13	Explore perimeter as an attribute of plane figures and solve problems. https://www.youtube.com/watch?v
	Lesson 14	Determine the perimeter of regular polygons and rectangles when whole number measurements are missing. https://www.youtube.com/watch?v
	Lesson 15	Solve word problems to determine perimeter with given side lengths. https://www.youtube.com/watch?v
	Lesson 16	Use string to measure the perimeter of various circles to the nearest quarter inch. https://www.youtube.com/watch?v
	Lesson 17	Use all four operations to solve problems involving perimeter and missing measurements.

	Mid Module Assessment	
	Dates are Flexible Due to PARCC Testing	
Topic D: Recording Perimeter and Area Data on Line Plots	Lesson 18	Construct rectangles from a given number of unit squares and determine the perimeters. https://www.youtube.com/watch?v
	Lesson 19	Use a line plot to record the number of rectangles constructed from a given number of unit squares. https://www.youtube.com/watch?v
	Lesson 20	Construct rectangles with a given perimeter using unit squares and determine their areas. https://www.youtube.com/watch?v
	Lesson 21	Construct rectangles with a given perimeter using unit squares and determine their areas. https://www.youtube.com/watch?v
	Lesson 22	Use a line plot to record the number of rectangles constructed in Lessons 20 and 21. https://www.youtube.com/watch?v
Topic E: Problem Solving with Perimeter and Area	Lesson 23	Solve a variety of word problems with perimeter. https://www.youtube.com/watch?v
	Lesson 24	Use rectangles to draw a robot with specified perimeter measurements, and reason about the different areas that may be produced. https://www.youtube.com/watch?v
	Lesson 25	Use rectangles to draw a robot with specified perimeter measurements, and reason about the different areas that may be produced. https://www.youtube.com/watch?v
	Lesson 26	Use rectangles to draw a robot with specified perimeter measurements, and reason about the different areas that may be produced. https://www.youtube.com/watch?v
	Lesson 27	Use rectangles to draw a robot with specified perimeter measurements, and reason about the different areas that may be produced. https://www.youtube.com/watch?v

	Lesson 28	Solve a variety of word problems involving area and perimeter using all four operations. https://www.youtube.com/watch?v
	Lesson 29	Solve a variety of word problems involving area and perimeter using all four operations. https://www.youtube.com/watch?v
	Lesson 30	Share and critique peer strategies for problem solving. https://www.youtube.com/watch?v
End Of Module Assessment		
June 17-19, 2019		
Topic F: Year In Review	Lesson 31	Explore and create unconventional representations of one-half.
	Lesson 32	Explore and create unconventional representations of one-half.
	Lesson 33	Solidify fluency with Grade 3 skills.
	Lesson 34	Create resource booklets to support fluency with Grade 3 skills.

NJSLS Standards:

3.OA.8	Solve two-step word problems using the four operations. Represent these problems using equations with a letter standing for the unknown quantity. Assess the reasonableness of answers using mental computation and estimation strategies including rounding.

- Students solve two-step problems that include more than one operation by representing the information using concrete models, pictures including bar models, and number lines. Writing equations begins with making connections between the representations and the symbolic notation (equations).

- Students should be exposed to multiple problem-solving strategies (using any combination of words, numbers, diagrams, physical objects or symbols) and be able to choose which ones to use that make most sense them.

Determining whether answers are reasonable by using number sense, understanding the context, the meaning of operations using mental computation strategies, and estimation strategies cannot be overemphasized as students work with all of their ideas embedded in this Standard.

- Using a letter standing for the unknown quantity should explicitly connect to previous work with identifying missing information that was represented by a box, underscore, or other symbols.

- When students solve word problems, they use various estimation skills which include identifying when estimation is appropriate, determining the level of accuracy needed, selecting the appropriate method of estimation, and verifying solutions or determining the reasonableness of solutions.

Example:
On a vacation, your family travels 267 miles on the first day, 194 miles on the second day and 34 miles on the third day. How many total miles did they travel?

3.MD.4	Generate measurement data by measuring lengths using rulers marked with halves and fourths of an inch. Show the data by making a line plot, where the horizontal scale is marked off in appropriate units—whole numbers, halves, or quarters.

- Show measurements on a line plot to display the information in an organized way.

Second Grade Heights
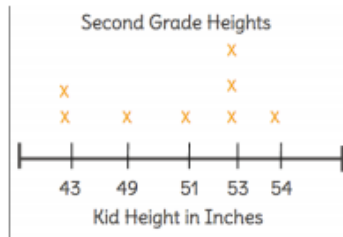
- Assure that students are accurately lining up the objects to be measure on the line plot and that the X's used are the same size to avoid misinterpretation of the data.

- Measure length using rulers marked with inch, quarter inch and half inch. Accurately measure several small objects using a standard ruler and display findings on a line plot. Third graders need many opportunities measuring the length of various objects in their environment.
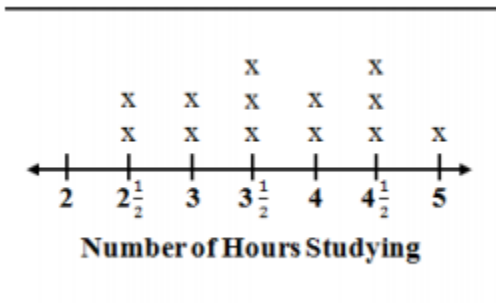
Example:
Measure objects in your desk to the nearest $\frac{1}{2}$ inch or $\frac{1}{4}$ of an inch.
Display data collected on a line plot.
How many objects measured $\frac{1}{2}$ inch? $\frac{1}{4}$ inch ?
Display data on line plots with horizontal scales in whole numbers, halves, and quarter.

Number of Hours Studying

Students should connect their understanding of fractions to the measuring of one-half and one-quarter inch.

3.MD.8	Solve real world and mathematical problems involving perimeters of polygons, including finding the perimeter given the side lengths, finding an unknown side length, and exhibiting rectangles with the same perimeter and different areas or with the same area and different perimeters.

- Measurement describes the attributes of objects and events. Standard units of measure enable people to interpret results or data

- Perimeter of a figure is equivalent to the sum of the length of all sides. Rectangles that have the same perimeter can have different areas. Rectangles that have same area can have

different perimeters.

- Relate addition and subtraction to length.

- Students develop an understanding of the concept of perimeter by walking around the perimeter of a room, using rubber bands to represent the perimeter of a plane figure on a geoboard, or tracing around a shape on an interactive whiteboard.

- Show rectangles that have the same perimeter but different areas. Show rectangles having different perimeters but the same area.

3.G.1	Understand that shapes in different categories (e.g., rhombuses, rectangles, and others) may share attributes (e.g., having four sides), and that the shared attributes can define a larger category (e.g., quadrilaterals). Recognize rhombuses, rectangles, and squares as examples of quadrilaterals, and draw examples of quadrilaterals that do not belong to any of these subcategories.

- Teacher promotes understanding of the structure of rectangular arrays and describing and analyzing two-dimensional shapes. Sets the tone and creates activities to help students investigate quadrilaterals (technology may be used during this exploration). Students recognize shapes that are and are not quadrilaterals by examining the properties of the geometric figures.

- Shapes in different categories share attributes.

- Quadrilaterals are figures with four sides.

- Students should be encouraged to provide details and use proper vocabulary when describing the properties of quadrilaterals. They sort geometric figures (see examples below) and identify squares, rectangles, and rhombuses as quadrilaterals.

- They conceptualize that a quadrilateral must be a closed figure with four straight sides and begin to notice characteristics of the angles and the relationship between opposite sides.

- Model vocabulary with numerous examples and encourage students to use geometric terms such as *properties, attributes, quadrilateral, open figure, closed figure, rhombus, rectangle,* and *square.*

Common multiplication and division situations. [1]

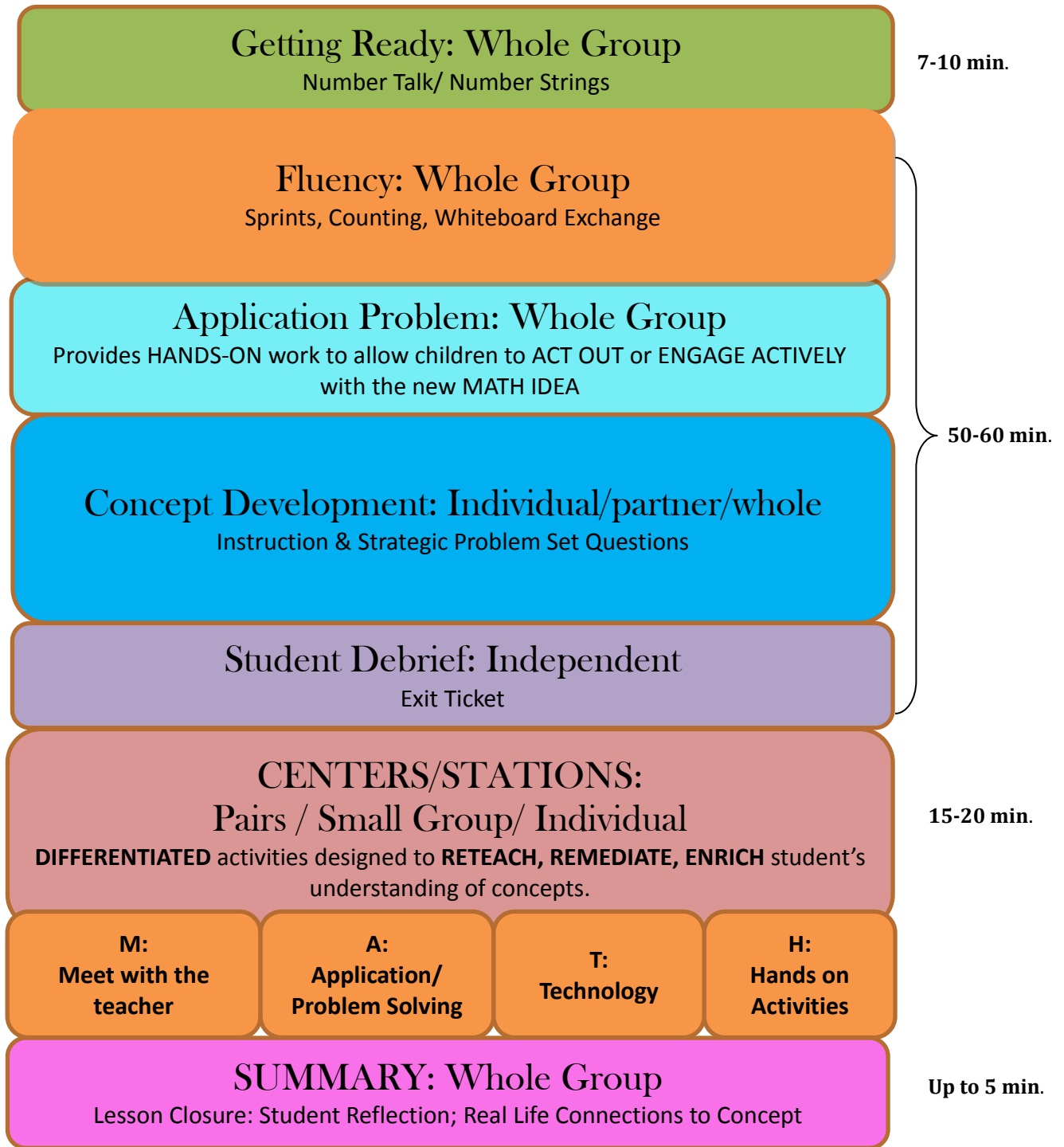|	UNKNOWN PRODUCT	GROUP SIZE UNKNOWN ("HOW MANY IN EACH GROUP?" DIVISION)	NUMBER OF GROUPS UNKNOWN ("HOW MANY GROUPS?" DIVISION)
	$3 \times 6 = ?$	$3 \times ? = 18$, and $18 \div 3 = ?$	$? \times 6 = 18$, and $18 \div 6 = ?$
EQUAL GROUPS	There are 3 bags with 6 plums in each bag. How many plums are there in all? *Measurement example.* You need 3 lengths of string, each 6 inches long. How much string will you need altogether?	If 18 plums are shared equally into 3 bags, then how many plums will be in each bag? *Measurement example.* You have 18 inches of string, which you will cut into 3 equal pieces. How long will each piece of string be?	If 18 plums are to be packed 6 to a bag, then how many bags are needed? *Measurement example.* You have 18 inches of string, which you will cut into pieces that are 6 inches long. How many pieces of string will you have?
ARRAYS [2], AREA [3]	There are 3 rows of apples with 6 apples in each row. How many apples are there? *Area example.* What is the area of a 3 cm by 6 cm rectangle?	If 18 apples are arranged into 3 equal rows, how many apples will be in each row? *Area example.* A rectangle has area 18 square centimeters. If one side is 3 cm long, how long is a side next to it?	If 18 apples are arranged into equal rows of 6 apples, how many rows will there be? *Area example.* A rectangle has area 18 square centimeters. If one side is 6 cm long, how long is a side next to it?
COMPARE	A blue hat costs $6. A red hat costs 3 times as much as the blue hat. How much does the red hat cost? *Measurement example.* A rubber band is 6 cm long. How long will the rubber band be when it is stretched to be 3 times as long?	A red hat costs $18 and that is 3 times as much as a blue hat costs. How much does a blue hat cost? *Measurement example.* A rubber band is stretched to be 18 cm long and that is 3 times as long as it was at first. How long was the rubber band at first?	A red hat costs $18 and a blue hat costs $6. How many times as much does the red hat cost as the blue hat? *Measurement example.* A rubber band was 6 cm long at first. Now it is stretched to be 18 cm long. How many times as long is the rubber band now as it was at first?
GENERAL	$a \times b = ?$	$a \times ? = p$ and $p \div a = ?$	$? \times b = p$, and $p \div b = ?$

[1] The language in the array examples shows the easiest form of array problems. A harder form is to use the terms rows and columns: The apples in the grocery window are in 3 rows and 6 columns. How many apples are in there? Both forms are valuable.

[2] Area involves arrays of squares that have been pushed together so that there are no gaps or overlaps, so array problems include these especially important measurement situations.

[3] The first examples in each cell are examples of discrete things. These are easier for students and should be given before the measurement examples.

Module 7 Assessment / Authentic Assessment Recommended Framework				
Assessment	**CCSS**	**Estimated Time**	**Format**
Eureka Math Module 7: *Geometry and Measurement Word Problems*			
Authentic Assessment: Classifying Shapes	3.G.1	30 mins	Individual
Authentic Assessment: Rectangular Garden	3.MD.8	30 mins	Individual
Optional Mid-Module Assessment	3.OA.8 3.MD.8 3.G.1	1 Block	Individual
Optional End of Module Assessment	3.OA.8 3.MD.4 3.MD.8 3.G.1	1 Block	Individual

Third Grade Ideal Math Block

Getting Ready: Whole Group
Number Talk/ Number Strings

7-10 min.

Fluency: Whole Group
Sprints, Counting, Whiteboard Exchange

Application Problem: Whole Group
Provides HANDS-ON work to allow children to ACT OUT or ENGAGE ACTIVELY with the new MATH IDEA

Concept Development: Individual/partner/whole
Instruction & Strategic Problem Set Questions

50-60 min.

Student Debrief: Independent
Exit Ticket

CENTERS/STATIONS:
Pairs / Small Group/ Individual
DIFFERENTIATED activities designed to RETEACH, REMEDIATE, ENRICH student's understanding of concepts.

15-20 min.

M: Meet with the teacher	A: Application/ Problem Solving	T: Technology	H: Hands on Activities

SUMMARY: Whole Group
Lesson Closure: Student Reflection; Real Life Connections to Concept

Up to 5 min.

Eureka Lesson Structure:

Fluency:

- Sprints

- Counting : Can start at numbers other than 0 or 1 and might include supportive concrete material or visual models

- Whiteboard Exchange

Application Problem:

- Engage students in using the RDW Process

- Sequence problems from simple to complex and adjust based on students' responses

- Facilitate share and critique of various explanations, representations, and/or examples.

Concept Development: (largest chunk of time)

Instruction:

- Maintain overall alignment with the objectives and suggested pacing and structure.

- Use of tools, precise mathematical language, and/or models

- Balance teacher talk with opportunities for peer share and/or collaboration

- Generate next steps by watching and listening for understanding

Problem Set: (Individual, partner, or group)

- Allow for independent practice and productive struggle

- Assign problems strategically to differentiate practice as needed

- Create and assign remedial sequences as needed

Student Debrief:

- Elicit students thinking, prompt reflection, and promote metacognition through student centered discussion

- Culminate with students' verbal articulation of their learning for the day

- Close with completion of the daily Exit Ticket (opportunity for informal assessment that guides effective preparation of subsequent lessons) as needed.

PARCC Assessment Evidence/Clarification Statements			
CCSS	**Evidence Statement**	**Clarification**	**MP**
3.OA.8	Solve two-step word problems using the four operations. Represent these problems using equations with a letter standing for the unknown quantity. Assess the reasonableness of answers using mental computation and estimation strategies including rounding	• Tasks do not require a student to write a single equation with a letter standing for the unknown quantity in a two-step problem, and then solve that equation. • Tasks may require students to write an equation as part of their work to find a solution, but students are not required to use a letter for the unknown. • Addition, subtraction, multiplication and division situations in these problems may involve any of the basic situation types with unknowns in various positions (see CCSSM, Table 1,Common addition and subtraction situations, p. 88; CCSSM, Table 2, Common multiplication and division situations, p. 89; and the document for the OA Progression).	MP 1,4
3.MD.4	Generate measurement data by measuring lengths using rulers marked with halves and fourths of an inch. Show the data by making a line plot, where the horizontal scale is marked off in appropriate units—whole numbers, halves, or quarters.		MP.2, MP.5
3.MD.8	Solve real world and mathematical problems involving perimeters of polygons, including finding the perimeter given the side lengths, finding an unknown side length, and exhibiting rectangles with the same perimeter and different areas or with the same area and different perimeters.		MP 2,4,5

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| 3.G.1	Understand that shapes in different categories (e.g., rhombuses, rectangles, and others) may share attributes (e.g., having four sides), and that the shared attributes can define a larger category (e.g., quadrilaterals). Recognize rhombuses, rectangles, and squares as examples of quadrilaterals, and draw examples of quadrilaterals that do not belong to any of these subcategories.		

Student Name: _____ Task: _____ School: _____ Teacher: _____ Date: _____

"I CAN....."	STUDENT FRIENDLY RUBRIC				SCORE
	...a start 1	...getting there 2	...that's it 3	WOW! 4	
Understand	I need help.	I need some help.	I do not need help.	I can help a class-mate.	
Solve	I am unable to use a strategy.	I can start to use a strategy.	I can solve it more than one way.	I can use more than one strategy and talk about how they get to the same answer.	
Say or Write	I am unable to say or write.	I can write or say some of what I did.	I can write and talk about what I did. I can write or talk about why I did it.	I can write and say what I did and why I did it.	
Draw or Show	I am not able to draw or show my thinking.	I can draw, but not show my thinking; or I can show but not draw my thinking;	I can draw and show my thinking	I can draw, show and talk about my think-ing.	

Use and Connection of Mathematical Representations
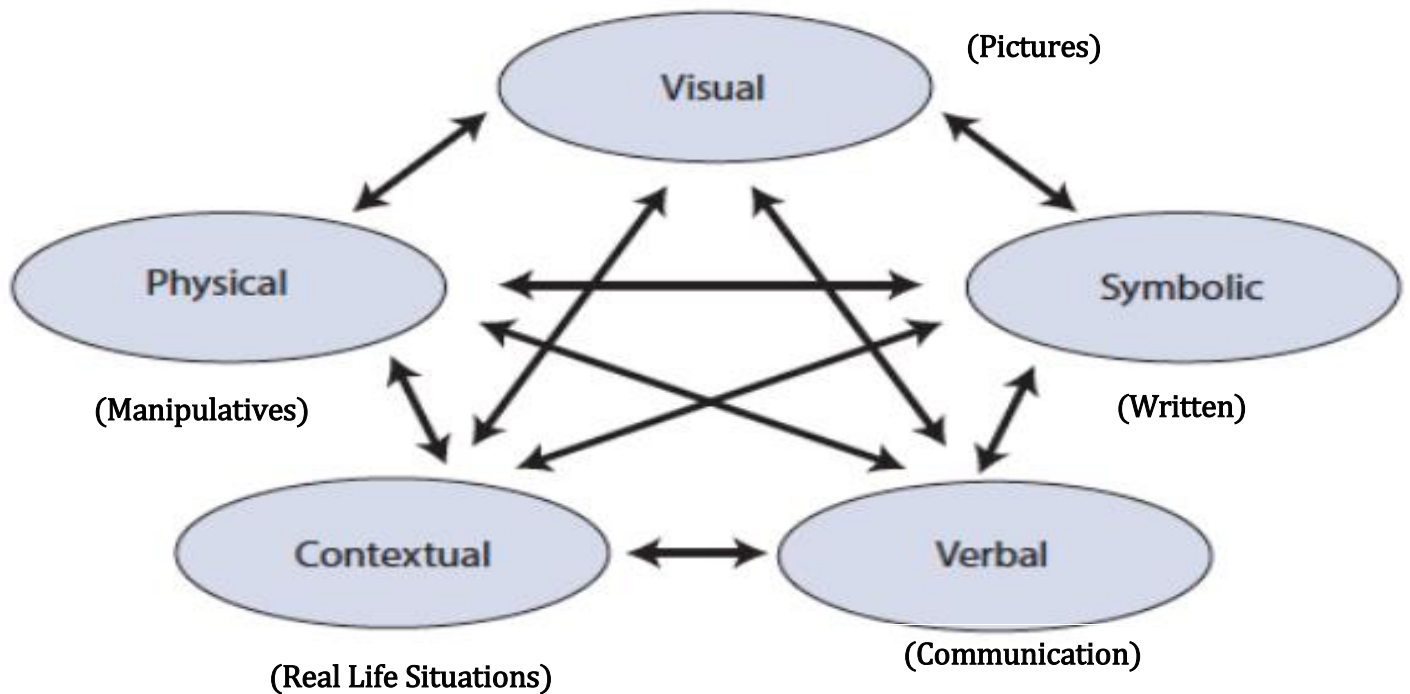
The Lesh Translation Model

Each oval in the model corresponds to one way to represent a mathematical idea.

Visual: When children draw pictures, the teacher can learn more about what they understand about a particular mathematical idea and can use the different pictures that children create to provoke a discussion about mathematical ideas. Constructing their own pictures can be a powerful learning experience for children because they must consider several aspects of mathematical ideas that are often assumed when pictures are pre-drawn for students.

Physical: The manipulatives representation refers to the unifix cubes, base-ten blocks, fraction circles, and the like, that a child might use to solve a problem. Because children can physically manipulate these objects, when used appropriately, they provide opportunities to compare relative sizes of objects, to identify patterns, as well as to put together representations of numbers in multiple ways.

Verbal: Traditionally, teachers often used the spoken language of mathematics but rarely gave students opportunities to grapple with it. Yet, when students do have opportunities to express their mathematical reasoning aloud, they may be able to make explicit some knowledge that was previously implicit for them.

Symbolic: Written symbols refer to both the mathematical symbols and the written words that are associated with them. For students, written symbols tend to be more abstract than the other representations. I tend to introduce symbols after students have had opportunities to make connections among the other representations, so that the students have multiple ways to connect the symbols to mathematical ideas, thus increasing the likelihood that the symbols will be comprehensible to students.

Contextual: A relevant situation can be any context that involves appropriate mathematical ideas and holds interest for children; it is often, but not necessarily, connected to a real-life situation.

The Lesh Translation Model: Importance of Connections

As important as the ovals are in this model, another feature of the model is even more important than the representations themselves: The arrows! The arrows are important because they represent the connections students make between the representations. When students make these connections, they may be better able to access information about a mathematical idea, because they have multiple ways to represent it and, thus, many points of access.

Individuals enhance or modify their knowledge by building on what they already know, so the greater the number of representations with which students have opportunities to engage, the more likely the teacher is to tap into a student's prior knowledge. This "tapping in" can then be used to connect students' experiences to those representations that are more abstract in nature (such as written symbols). Not all students have the same set of prior experiences and knowledge. Teachers can introduce multiple representations in a meaningful way so that students' opportunities to grapple with mathematical ideas are greater than if their teachers used only one or two representations.

Concrete Pictorial Abstract (CPA) Instructional Approach

The CPA approach suggests that there are three steps necessary for pupils to develop understanding of a mathematical concept.

Concrete: "Doing Stage": Physical manipulation of objects to solve math problems.

Pictorial: "Seeing Stage": Use of imaged to represent objects when solving math problems.

Abstract: "Symbolic Stage": Use of only numbers and symbols to solve math problems.

CPA is a gradual systematic approach. Each stage builds on to the previous stage. Reinforcement of concepts are achieved by going back and forth between these representations and making connections between stages. Students will benefit from seeing parallel samples of each stage and how they transition from one to another.

Read, Draw, Write Process

READ the problem. Read it over and over…. And then read it again.

DRAW a picture that represents the information given. During this step students ask themselves: Can I draw something from this information? What can I draw? What is the best model to show the information? What conclusions can I make from the drawing?

WRITE your conclusions based on the drawings. This can be in the form of a number sentence, an equation, or a statement.

Students are able to draw a model of what they are reading to help them understand the problem. Drawing a model helps students see which operation or operations are needed, what patterns might arise, and which models work and do not work. Students must dive deeper into the problem by drawing models and determining which models are appropriate for the situation.

While students are employing the RDW process they are using several Standards for Mathematical Practice and in some cases, all of them.

Mathematical Discourse and Strategic Questioning

Discourse involves asking strategic questions that elicit from students their understanding of the context and actions taking place in a problem, how a problem is solved and why a particular method was chosen. Students learn to critique their own and others' ideas and seek out efficient mathematical solutions.

While classroom discussions are nothing new, the theory behind classroom discourse stems from constructivist views of learning where knowledge is created internally through interaction with the environment. It also fits in with socio-cultural views on learning where students working together are able to reach new understandings that could not be achieved if they were working alone.

Underlying the use of discourse in the mathematics classroom is the idea that mathematics is primarily about reasoning not memorization. Mathematics is not about remembering and applying a set of procedures but about developing understanding and explaining the processes used to arrive at solutions.

Teacher Questioning:

Asking better questions can open new doors for students, promoting mathematical thinking and classroom discourse. Can the questions you're asking in the mathematics classroom be answered with a simple "yes" or "no," or do they invite students to deepen their understanding?

The most important thing is to NEVER stop questioning

Albert Einstein

To help you encourage deeper discussions, here are 100 questions to incorporate into your instruction by Dr. Gladis Kersaint, mathematics expert and advisor for Ready Mathematics.

Help students learn to reason mathematically

26 How did you begin to think about this problem?

27 What is another way you could solve this problem?

28 How could you prove _____?

29 Can you explain how your answer is different from or the same as [student]'s answer?

30 Let's break the problem into parts. What would the parts be?

31 Can you explain this part more specifically?

32 Does that always work?

33 Can you think of a case where that wouldn't work?

34 How did you organize your information? Your thinking?

Help students evaluate their own processes and engage in productive peer interaction

35 What do you need to do next?

36 What have you accomplished?

37 What are your strengths and weaknesses?

38 Was your group participation appropriate and helpful?

Help students with problem comprehension

39 What is this problem about? What can you tell me about it?

40 Do you need to define or set limits for the problem?

41 How would you interpret that?

42 Could you reword that in simpler terms?

43 Is there something that can be eliminated or that is missing?

44 Could you explain what the problem is asking?

45 What assumptions do you have to make?

46 What do you know about this part?

47 Which words were most important? Why?

Help students learn to **conjecture, invent, and solve** problems

48 **What would happen if ___?**

49 Do you see a **pattern**?

50 What are some **possibilities** here?

51 Where could you find the **information** you need?

52 How would you **check your steps** or your answer?

53 What **did not work**?

54 How is your solution method the **same as or different from** [student]'s method?

55 Other than retracing your steps, **how can you determine** if your answers are appropriate?

56 How did you **organize** the information? Do you have a **record**?

57 How could you solve this using **tables, lists, pictures, diagrams**, etc.?

58 What have you tried? What **steps** did you take?

59 How would it look if you used this **model** or these **materials**?

60 How would you draw a **diagram or make a sketch** to solve the problem?

61 Is there **another possible answer**? If so, explain.

62 Is there **another way to solve** the problem?

63 Is there **another model** you could use to solve the problem?

64 Is there anything you've **overlooked**?

65 **How did you think** about the problem?

66 What was your **estimate or prediction**?

67 How **confident** are you in your answer?

68 **What else** would you like to know?

69 What do you think comes **next**?

70 Is the solution **reasonable**, considering the context?

71 Did you have a **system**? Explain it.

72 Did you have a **strategy**? Explain it.

73 Did you have a **design**? Explain it.

Help students learn to **connect mathematics, its ideas, and its application**

74 What is the **relationship** between ___ and ___?

75 Have we ever solved a problem **like this before**?

76 What uses of mathematics did you find in the **newspaper** last night?

77 What is the **same**?

78 What is **different**?

79 Did you use skills or build on concepts that were **not necessarily mathematical**?

80 Which **skills or concepts** did you use?

81 What **ideas** have we explored before that were useful in solving this problem?

82 Is there a **pattern**?

83 **Where else** would this strategy be useful?

84 How does this **relate** to ___?

85 Is there a **general rule**?

86 Is there a **real-life situation** where this could be used?

87 How would your method work with **other problems**?

88 What other problem does this seem to **lead to**?

Help students **persevere**

89 Have you tried making a **guess**?

90 **What else** have you tried?

91 Would **another method** work as well or better?

92 Is there **another way** to draw, explain, or say that?

93 Give me another **related problem**. Is there an easier problem?

94 How would you **explain** what you know right now?

95 What was **one thing you learned** (or two, or more)?

96 Did you **notice any patterns**? If so, describe them.

97 What **mathematics topics** were used in this investigation?

98 What were the **mathematical ideas** in this problem?

99 What is mathematically **different about these two situations**?

100 What are the **variables** in this problem? What stays **constant**?

Help students focus on the mathematics from **activities**

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Conceptual Understanding

Students demonstrate conceptual understanding in mathematics when they provide evidence that they can:
- recognize, label, and generate examples of concepts;
- use and interrelate models, diagrams, manipulatives, and varied representations of concepts;
- identify and apply principles; know and apply facts and definitions;
- compare, contrast, and integrate related concepts and principles; and
- recognize, interpret, and apply the signs, symbols, and terms used to represent concepts.

Conceptual understanding reflects a student's ability to reason in settings involving the careful application of concept definitions, relations, or representations of either.

Procedural Fluency

Procedural fluency is the ability to:
- apply procedures accurately, efficiently, and flexibly;
- to transfer procedures to different problems and contexts;
- to build or modify procedures from other procedures; and
- to recognize when one strategy or procedure is more appropriate to apply than another.

Procedural fluency is more than memorizing facts or procedures, and it is more than understanding and being able to use one procedure for a given situation. Procedural fluency builds on a foundation of conceptual understanding, strategic reasoning, and problem solving (NGA Center & CCSSO, 2010; NCTM, 2000, 2014). Research suggests that once students have memorized and practiced procedures that they do not understand, they have less motivation to understand their meaning or the reasoning behind them (Hiebert, 1999). Therefore, the development of students' conceptual understanding of procedures should precede and coincide with instruction on procedures.

Math Fact Fluency: Automaticity

Students who possess math fact fluency can recall math facts with automaticity. Automaticity is the ability to do things without occupying the mind with the low-level details required, allowing it to become an automatic response pattern or habit. It is usually the result of learning, repetition, and practice.

3-5 Math Fact Fluency Expectation
3.OA.C.7: Single-digit products and quotients (Products from memory by end of Grade 3)
3.NBT.A.2: Add/subtract within 1000
4.NBT.B.4: Add/subtract within 1,000,000/ Use of Standard Algorithm
5.NBT.B.5: Multi-digit multiplication/ Use of Standard Algorithm

Evidence of Student Thinking

Effective classroom instruction and more importantly, improving student performance, can be accomplished when educators know how to elicit evidence of students' understanding on a daily basis. Informal and formal methods of collecting evidence of student understanding enable educators to make positive instructional changes. An educators' ability to understand the processes that students use helps them to adapt instruction allowing for student exposure to a multitude of instructional approaches, resulting in higher achievement. By highlighting student thinking and misconceptions, and eliciting information from more students, all teachers can collect more representative evidence and can therefore better plan instruction based on the current understanding of the entire class.

Mathematical Proficiency

To be mathematically proficient, a student must have:

- Conceptual understanding: comprehension of mathematical concepts, operations, and relations;

- Procedural fluency: skill in carrying out procedures flexibly, accurately, efficiently, and appropriately;

- Strategic competence: ability to formulate, represent, and solve mathematical problems;

- Adaptive reasoning: capacity for logical thought, reflection, explanation, and justification;

- Productive disposition: habitual inclination to see mathematics as sensible, useful, and worthwhile, coupled with a belief in diligence and one's own efficacy.

Evidence should:

- Provide a window in student thinking;

- Help teachers to determine the extent to which students are reaching the math learning goals; and

- Be used to make instructional decisions during the lesson and to prepare for subsequent lessons.

Formative assessment is an essentially interactive process, in which the teacher can find out whether what has been taught has been learned, and if not, to do something about it. Day-to-day formative assessment is one of the most powerful ways of improving learning in the mathematics classroom.

(Wiliam 2007, pp. 1054; 1091)
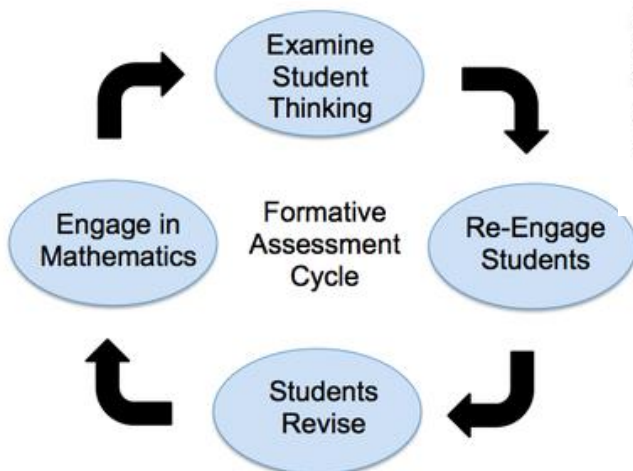
Connections to the Mathematical Practices

Student Friendly Connections to the Mathematical Practices

1. I can solve problems without giving up.
2. I can think about numbers in many ways.
3. I can explain my thinking and try to understand others.
4. I can show my work in many ways.
5. I can use math tools and tell why I choose them.
6. I can work carefully and check my work.
7. I can use what I know to solve new problems.
8. I can discover and use short cuts.

The Standards for Mathematical Practice:
Describe varieties of expertise that mathematics educators at all levels should seek to develop in their students.

	Make sense of problems and persevere in solving them
1	In **third** grade, students know that doing mathematics involves solving problems and discussing how they solved them. Students explain to themselves the meaning of a problem and look for ways to solve it. Third graders may use concrete objects or pictures to help them conceptualize and solve problems. They may check their thinking by asking themselves, "Does this make sense?" They listen to the strategies of others and will try approaches. They often will use another method to check their answers.
	Reason abstractly and quantitatively
2	In **third** grade, students should recognize that number represents a specific quantity. They connect quantity to written symbols and create logical representation of the problem at hand, considering both the appropriate units involved and the meaning of quantities
	Construct viable arguments and critique the reasoning of others
3	In **third** grade, mathematically proficient students may construct viable arguments using concrete referents, such as objects, pictures, and drawings. They refine their mathematical communication skills as they participate in mathematical discussions involving questions like, "How did you get that?" and "Why is it true?" They explain their thinking to others and respond to others' thinking.
	Model with mathematics
4	Mathematically proficient students experiment with representing problem situations in multiple ways including numbers, words (mathematical language) drawing pictures, using objects, acting out, making chart, list, or graph, creating equations etc…Students need opportunities to connect different representations and explain the connections. They should be able to use all of the representations as needed. **Third** graders should evaluate their results in the context of the situation and reflect whether the results make any sense.
	Use appropriate tools strategically
5	**Third** graders should consider all the available tools (including estimation) when solving a mathematical problem and decide when certain tools might be helpful. For example, they might use graph paper to find all possible rectangles with the given perimeter. They compile all possibilities

	into an organized list or a table, and determine whether they all have the possible rectangles.
6	**Attend to precision** Mathematical proficient **third** graders develop their mathematical communication skills; they try to use clear and precise language in their discussions with others and in their own reasoning. They are careful about specifying their units of measure and state the meaning of the symbols they choose. For instance, when figuring out the area of a rectangle the record their answer in square units.
7	**Look for and make use of structure** In **third** grade, students should look closely to discover a pattern of structure. For example, students properties of operations as strategies to multiply and divide. (commutative and distributive properties.
8	**Look for and express regularity in repeated reasoning** Mathematically proficient students in third **grade** should notice repetitive actions in computation and look for more shortcut methods. For example, students may use the distributive property as a strategy for using products they know to solve products that they don't know. For example, if students are asked to find the product of 7x8, they might decompose 7 into 5 and 2 and then multiply 5 x 8 and 2 x 8 to arrive at 40 + 16 or 56. In addition, third graders continually evaluate their work by asking themselves, "Does this make sense?"

Effective Mathematics Teaching Practices

Establish mathematics goals to focus learning. Effective teaching of mathematics establishes clear goals for the mathematics that students are learning, situates goals within learning progressions, and uses the goals to guide instructional decisions.

Implement tasks that promote reasoning and problem solving. Effective teaching of mathematics engages students in solving and discussing tasks that promote mathematical reasoning and problem solving and allow multiple entry points and varied solution strategies.

Use and connect mathematical representations. Effective teaching of mathematics engages students in making connections among mathematical representations to deepen understanding of mathematics concepts and procedures and as tools for problem solving.

Facilitate meaningful mathematical discourse. Effective teaching of mathematics facilitates discourse among students to build shared understanding of mathematical ideas by analyzing and comparing student approaches and arguments.

Pose purposeful questions. Effective teaching of mathematics uses purposeful questions to assess and advance students' reasoning and sense making about important mathematical ideas and relationships.

Build procedural fluency from conceptual understanding. Effective teaching of mathematics builds fluency with procedures on a foundation of conceptual understanding so that students, over time, become skillful in using procedures flexibly as they solve contextual and mathematical problems.

Support productive struggle in learning mathematics. Effective teaching of mathematics consistently provides students, individually and collectively, with opportunities and supports to engage in productive struggle as they grapple with mathematical ideas and relationships.

Elicit and use evidence of student thinking. Effective teaching of mathematics uses evidence of student thinking to assess progress toward mathematical understanding and to adjust instruction continually in ways that support and extend learning.

5 Practices for Orchestrating Productive Mathematics Discussions	
Practice	Description/ Questions
1. Anticipating	What strategies are students likely to use to approach or solve a challenging high-level mathematical task? How do you respond to the work that students are likely to produce? Which strategies from student work will be most useful in addressing the mathematical goals?
2. Monitoring	Paying attention to what and how students are thinking during the lesson. Students working in pairs or groups Listening to and making note of what students are discussing and the strategies they are using Asking students questions that will help them stay on track or help them think more deeply about the task. (Promote productive struggle)
3. Selecting	This is the process of deciding the *what* and the *who* to focus on during the discussion.
4. Sequencing	What order will the solutions be shared with the class?
5. Connecting	Asking the questions that will make the mathematics explicit and understandable. Focus must be on mathematical meaning and relationships; making links between mathematical ideas and representations.

MATH CENTERS/ WORKSTATIONS

Math workstations allow students to engage in authentic and meaningful hands-on learning. They often last for several weeks, giving students time to reinforce or extend their prior instruction. Before students have an opportunity to use the materials in a station, introduce them to the whole class, several times. Once they have an understanding of the concept, the materials are then added to the work stations.

Station Organization and Management Sample

Teacher A has 12 containers labeled 1 to 12. The numbers correspond to the numbers on the rotation chart. She pairs students who can work well together, who have similar skills, and who need more practice on the same concepts or skills. Each day during math work stations, students use the center chart to see which box they will be using and who their partner will be. Everything they need for their station will be in their box. **Each station is differentiated**. If students need more practice and experience working on numbers 0 to 10, those will be the only numbers in their box. If they are ready to move on into the teens, then she will place higher number activities into the box for them to work with.

In the beginning there is a lot of prepping involved in gathering, creating, and organizing the work stations. However, once all of the initial work is complete, the stations are easy to manage. Many of her stations stay in rotation for three or four weeks to give students ample opportunity to master the skills and concepts.

Read *Math Work Stations* by Debbie Diller.
In her book, she leads you step-by-step through the process of implementing work stations.

MATH WORKSTATION INFORMATION CARD

Math Workstation: _____   Time: _____

NJSLS.:

_____
_____
_____
_____

Objective(s): By the end of this task, I will be able to:

- _____
- _____
- _____

Task(s):

- _____
- _____
- _____
- _____

Exit Ticket:

- _____
- _____
- _____

MATH WORKSTATION SCHEDULE   Week of: _____

DAY	Technology Lab	Problem Solving Lab	Fluency Lab	Math Journal	Small Group Instruction
Mon.	Group ____	Group ____	Group ____	Group ____	BASED ON CURRENT OBSERVATIONAL DATA
Tues.	Group ____	Group ____	Group ____	Group ____	
Wed.	Group ____	Group ____	Group ____	Group ____	
Thurs.	Group ____	Group ____	Group ____	Group ____	
Fri.	Group ____	Group ____	Group ____	Group ____	

INSTRUCTIONAL GROUPING

	GROUP A		GROUP B
1		1	
2		2	
3		3	
4		4	
5		5	
6		6	

	GROUP C		GROUP D
1		1	
2		2	
3		3	
4		4	
5		5	
6		6	

Got It		Not There Yet		
Evidence shows that the student essentially has the target concept or big math idea.		Student shows evidence of a major misunderstanding, incorrect concepts or procedure, or a failure to engage in the task.		
PLD Level 5: 100% Distinguished command	PLD Level 4: 89% Strong Command	PLD Level 3: 79% Moderate Command	PLD Level 2: 69% Partial Command	PLD Level 1: 59% Little Command
Student work shows **distinguished levels of understanding** of the mathematics. Student **constructs** and **communicates** a **complete response** based on explanations/reasoning using the: • Tools: 　o Manipulatives 　o Five Frame 　o Ten Frame 　o Number Line 　o Part-Part-Whole Model • Strategies: 　o Drawings 　o Counting All 　o Count On/Back 　o Skip Counting 　o Making Ten 　o Decomposing Number • Precise use of math vocabulary Response includes an **efficient and logical progression** of mathematical reasoning and understanding.	Student work shows **strong levels of understanding** of the mathematics. Student **constructs** and **communicates** a **complete response** based on explanations/reasoning using the: • Tools: 　o Manipulatives 　o Five Frame 　o Ten Frame 　o Number Line 　o Part-Part-Whole Model • Strategies: 　o Drawings 　o Counting All 　o Count On/Back 　o Skip Counting 　o Making Ten 　o Decomposing Number • Precise use of math vocabulary Response includes a **logical progression** of mathematical reasoning and understanding.	Student work shows **moderate levels of understanding** of the mathematics. Student **constructs** and **communicates** a **complete response** based on explanations/reasoning using the: • Tools: 　o Manipulatives 　o Five Frame 　o Ten Frame 　o Number Line 　o Part-Part-Whole Model • Strategies: 　o Drawings 　o Counting All 　o Count On/Back 　o Skip Counting 　o Making Ten 　o Decomposing Number • Precise use of math vocabulary Response includes a **logical but incomplete progression** of mathematical reasoning and understanding. Contains **minor errors**.	Student work shows **partial understanding** of the mathematics. Student **constructs** and **communicates** an **incomplete response** based on student's attempts of explanations/ reasoning using the: • Tools: 　o Manipulatives 　o Five Frame 　o Ten Frame 　o Number Line 　o Part-Part-Whole Model • Strategies: 　o Drawings 　o Counting All 　o Count On/Back 　o Skip Counting 　o Making Ten 　o Decomposing Number • Precise use of math vocabulary Response includes an **incomplete or illogical progression** of mathematical reasoning and understanding.	Student work shows **little understanding** of the mathematics. Student **attempts** to **constructs** and **communicates** a response using the: • Tools: 　o Manipulatives 　o Five Frame 　o Ten Frame 　o Number Line 　o Part-Part-Whole Model • Strategies: 　o Drawings 　o Counting All 　o Count On/Back 　o Skip Counting 　o Making Ten 　o Decomposing Number • Precise use of math vocabulary Response includes **limited evidence of the progression** of mathematical reasoning and understanding.
5 points	4 points	3 points	2 points	1 point

DATA DRIVEN INSTRUCTION

Formative assessments inform instructional decisions. Taking inventories and assessments, observing reading and writing behaviors, studying work samples and listening to student talk are essential components of gathering data. When we take notes, ask questions in a student conference, lean in while a student is working or utilize a more formal assessment we are gathering data. Learning how to take the data and record it in a meaningful way is the beginning of the cycle.

Analysis of the data is an important step in the process. What is this data telling us? We must look for patterns, as well as compare the notes we have taken with work samples and other assessments. We need to decide what are the strengths and needs of individuals, small groups of students and the entire class. Sometimes it helps to work with others at your grade level to analyze the data.

Once we have analyzed our data and created our findings, it is time to make informed instructional decisions. These decisions are guided by the following questions:

- What mathematical practice(s) and strategies will I utilize to teach to these needs?
- What sort of grouping will allow for the best opportunity for the students to learn what it is I see as a need?
- Will I teach these strategies to the whole class, in a small guided group or in an individual conference?
- Which method and grouping will be the most effective and efficient? What specific objective(s) will I be teaching?

Answering these questions will help inform instructional decisions and will influence lesson planning.

Then we create our instructional plan for the unit/month/week/day and specific lessons.

It's important now to reflect on what you have taught.

Did you observe evidence of student learning through your checks for understanding, and through direct application in student work?

What did you hear and see students doing in their reading and writing?

Now it is time to begin the analysis again.
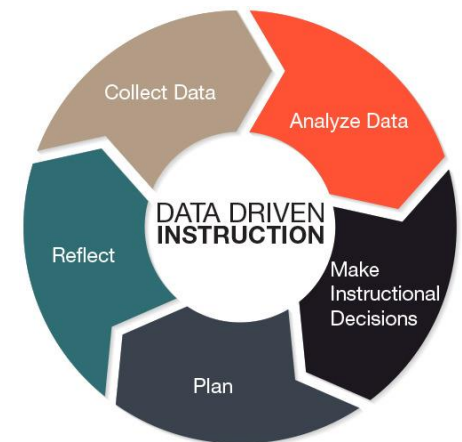
Data Analysis Form

School: _____   Teacher: _____   Date: _____

Assessment: _____   NJSLS: _____

GROUPS (STUDENT INITIALS)	SUPPORT PLAN	PROGRESS
MASTERED (86% - 100%) (PLD 4/5):		
DEVELOPING (67% - 85%) (PLD 3):		
INSECURE (51%-65%) (PLD 2):		
BEGINNING (0%-50%) (PLD 1):		

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MATH PORTFOLIO EXPECTATIONS

The Student Assessment Portfolios for Mathematics are used as a means of documenting and evaluating students' academic growth and development over time and in relation to the CCSS-M. The September task entry(-ies) should reflect the prior year content and *can serve* as an additional baseline measure.

All tasks contained within the **Student Assessment Portfolios** should be aligned to NJSLS and be "practice forward" (closely aligned to the Standards for Mathematical Practice).

Four (4) or more additional tasks will be included in the **Student Assessment Portfolios** for Student Reflection and will be labeled as such.

K-2 GENERAL PORTFOLIO EXPECTATIONS:

- Tasks contained within the Student Assessment Portfolios are "practice forward" and denoted as "Individual", "Partner/Group", and "Individual w/Opportunity for Student Interviews[1].

- Each Student Assessment Portfolio should contain a "Task Log" that documents all tasks, standards, and rubric scores aligned to the performance level descriptors (PLDs).

- Student work should be attached to a completed rubric; with appropriate teacher feedback on student work.

- Students will have multiple opportunities to revisit certain standards. Teachers will capture each additional opportunity "as a new and separate score" in the task log.

- A 2-pocket folder for each Student Assessment Portfolio is *recommended.*

- All Student Assessment Portfolio entries should be scored and recorded as an Authentic Assessment grade (25%)[2].

- All Student Assessment Portfolios must be clearly labeled, maintained for all students, inclusive of constructive teacher and student feedback and accessible for review.

GRADES K-2

Student Portfolio Review

Provide students the opportunity to review and evaluate their portfolio at various points throughout the year; celebrating their progress and possibly setting goals for future growth. During this process, students should retain ALL of their current artifacts in their Mathematics Portfolio

• For each shape pictured in the table, select the box for each correct shape name. You may select more than one box for each shape. If the shape is not a rectangle, rhombus, or quadrilateral, select the box for None of These.

Shape	Rectangle	Rhombus	Quadrilateral	None of These
▭	☐	☐	☐	☐
⬠	☐	☐	☐	☐
▢	☐	☐	☐	☐
▱	☐	☐	☐	☐
◇	☐	☐	☐	☐

Shape	Rectangle	Rhombus	Quadrilateral	None of These
▭	☑	☐	☑	☐
⬠	☐	☐	☐	☑
▢	☑	☑	☑	☐
⏢	☐	☐	☑	☐
◇	☐	☑	☑	☐

3.G.1

No Command	Partial Accomplishment	Substantial Accomplishment	Complete Mastery
0	**1**	**2**	**3**
All is incorrect	Student checks all the correct boxes for 1 shape.	Student checks all the correct boxes for 2-3 shapes.	Student checks all the correct boxes for 4-5 shapes.

Part A

The rectangular garden at River Valley School is represented in the figure. The perimeter of the garden is 122 yards.

21 yards

? yards
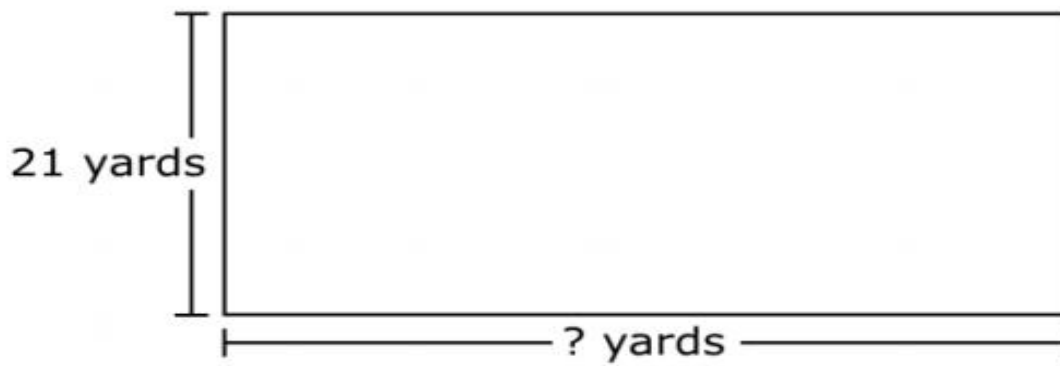
What is the missing side length in the figure?

Part B

River Valley School builds a new rectangular garden with the same perimeter, but the side lengths are different. What could be the side lengths of the new garden?

Part A	
Score	**Description**

1	Student response includes the following element:
	
	    • **Computation component:** 1 point
	
	       40 yards

0	Student response is incorrect or irrelevant

Part B	
Score	**Description**

1	Student response includes the following element
	
	    • **Side lengths:** 1 point
	
	       Ex: 14 yards by 47 yards
	
	       Accept any answer that provides different side lengths, yet has the same perimeter of 122 yards for the rectangular garden.

0	Student reponse is incorrect or irrelevant.

Resources

Engage NY
http://www.engageny.org/video-library?f[0]=im_field_subject%3A19

Common Core Tools
http://commoncoretools.me/
http://www.ccsstoolbox.com/
http://www.achievethecore.org/steal-these-tools

Achieve the Core
http://achievethecore.org/dashboard/300/search/6/1/0/1/2/3/4/5/6/7/8/9/10/11/12

Manipulatives
http://nlvm.usu.edu/en/nav/vlibrary.html

http://www.explorelearning.com/index.cfm?method=cResource.dspBrowseCorrelations&v=s&id=USA-000

http://www.thinkingblocks.com/

Illustrative Math Project :http://illustrativemathematics.org/standards/k8

Inside Mathematics: http://www.insidemathematics.org/index.php/tools-for-teachers

Sample Balance Math Tasks: http://www.nottingham.ac.uk/~ttzedweb/MARS/tasks/

Georgia Department of Education:https://www.georgiastandards.org/Common-Core/Pages/Math-K-5.aspx

Gates Foundations Tasks:http://www.gatesfoundation.org/college-ready-education/Documents/supporting-instruction-cards-math.pdf

Minnesota STEM Teachers' Center:
http://www.scimathmn.org/stemtc/frameworks/721-proportional-relationships

Singapore Math Tests K-12: http://www.misskoh.com

Mobymax.com: http://www.mobymax.com

21st Century Career Ready Practices

CRP1. Act as a responsible and contributing citizen and employee.
CRP2. Apply appropriate academic and technical skills.
CRP3. Attend to personal health and financial well-being.
CRP4. Communicate clearly and effectively and with reason.
CRP5. Consider the environmental, social and economic impacts of decisions.
CRP6. Demonstrate creativity and innovation.
CRP7. Employ valid and reliable research strategies.
CRP8. Utilize critical thinking to make sense of problems and persevere in solving them.
CRP9. Model integrity, ethical leadership and effective management.
CRP10. Plan education and career paths aligned to personal goals.
CRP11. Use technology to enhance productivity.
CRP12. Work productively in teams while using cultural global competence.

For additional details see **21st Century Career Ready Practices** .

References

"Eureka Math" *Great Minds.* 2018 < https://greatminds.org/account/products>